1. Introduction

Lift-and-project resembles a Swiss army knife to define and generate valid cuts.

But why using just one blade to cut in a single way? Perhaps we could avoid getting increasingly more parallel and shallow cuts by diversifying in the CGLP.

2. Background

Consider a binary linear program

\[
\min \{c^T x : A x \geq b, x \in \{0,1\}^n\}
\]

with linear relaxation

\[
P := \{x : A x \geq b, 0 \leq x \leq 1\}.
\]

If \(P\) is found in the \(\sigma\)-projection of \(P\) for \(\sigma > 0\), and it has as dual the lift-and-project description of \(x \in P\) as \(x = x^d + x^f\) for

\[
\begin{align*}
\alpha - u^A - u^e x^d &= 0, \\
\alpha - v^A - v^e x^f &= 0, \\
\beta - u^b &\leq 0, \\
\beta - v^b - v^f x^f &= 0, \\
u^b &= 0, \\
v^b &= 0.
\end{align*}
\]

3. Reverse polar CGLP

For some \(p \in P\), we propose the following formulation:

\[
\min \left\{ \alpha p - \beta : \gamma \geq 1 \right\}
\]

s.t. \((C)\)

\[
\begin{align*}
\beta - \alpha^T x &= 1, \\
\sum_{j \in J} \gamma_j &= \gamma, \\
\gamma_j &\geq 0, \\
\gamma_j &\leq 0, \\
\end{align*}
\]

Using \((C)\), instead, (RP-CGLP) is an extended formulation of the reverse polar \((P^\perp - x) := (y : y^T (x - \bar{x}) \geq 1 \forall x \in P)\).

Following the reverse polar notation, we denote cuts in the form \(\gamma_j (x - \bar{x}) \geq 1\).

\[\bullet\] Cuts from (RP-CGLP) define supporting hyperplanes of \(P^\perp\).

Among parallel inequalities, getting closer to \(\rho\) reduces \(y\) since distance to \(\bar{x}\) cut norm \(1\), thus reducing \(y^T (x - \bar{x})\).

\[\bullet\] No cutting plane is strictly dominated

How facets of \(P^\perp\) are combined in cutting planes from (RP-CGLP)?

Let the facet-defining inequalities of \(P^\perp\) define \(F = \{(y^T (x - \bar{x}) \geq \delta_{j,w} | \bar{x} \in F\}\) be finite for \(A, b, r, \nu\).

We let partition \(F = F^\perp \cup \nu F^\perp F^\nu\), where:

\[
\delta_j = \begin{cases} 
1, & j \in F^\perp \\
0, & j \in F^\nu
\end{cases}
\]

4. Equivalent formulations

A similar reformulation is proposed by Balas and Perregaard [2002],

\[
\min \left\{ \alpha^T x - \beta : \gamma \geq 1 \right\}
\]

s.t. \((C)\)

\[
\begin{align*}
\alpha - u^A - u^e x^d &= 0, \\
\alpha - v^A - v^e x^f &= 0, \\
\beta - u^b &\leq 0, \\
\beta - v^b - v^f x^f &= 0, \\
u^b &= 0, \\
v^b &= 0.
\end{align*}
\]

with the nonnegativity of \(x^d, x^f \geq 0\) relaxed (Cerasio and Soares, 1997).

We look for \(\sigma^A > 0\) separating \(x\) with a Cut Generating Linear Program (CGLP)

minimizing \(\alpha^T x - \beta\) and bounding (CGLP):

\[
\min \left\{ \alpha^T x - \beta \right\}
\]

s.t. \((C)\)

\[
\begin{align*}
\alpha^T x - \beta &= 1, \\
\alpha^T x - \beta &= 1, \\
\alpha^T x - \beta &= 1, \\
\alpha^T x - \beta &= 1.
\end{align*}
\]

5. Finding \(p\)

\[\bullet\] Any proper convex combination of points in the relative interior of each term of \(\bar{D}\) is in the relative interior of \(\bar{P}\).

\[\bullet\] Any point in the relative interior of \(\bar{P}\) has a convex combination of points in the relative interior of each \(\bar{D}\).

For a split on \(j = 1\), (CGLP) yields:

\[
(\text{c1) if } k \leq 8: (\text{c2) if } k = 8: (\text{c3) if } k \geq 8).
\]

7. Next steps

Beyond finding just some \(p \in \text{int}(P)\):

\[\bullet\] find \(p\) yielding one, facet-defining cut

\[\bullet\] find such points yielding disjoint cuts

6. Results

Common polar equality in the first round and better performance in the second

Gap closed (%) by adding split cuts on fractional variables with standard (S) and RP CGLP (R) and resolving on MIPLIB:

<table>
<thead>
<tr>
<th>1 round</th>
<th>2 rounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>R</td>
</tr>
<tr>
<td>p0032</td>
<td>50.5</td>
</tr>
<tr>
<td>p0033</td>
<td>26.2</td>
</tr>
<tr>
<td>p0104</td>
<td>6.7</td>
</tr>
<tr>
<td>p0301</td>
<td>0.0</td>
</tr>
<tr>
<td>p0302</td>
<td>84.4</td>
</tr>
<tr>
<td>p0932</td>
<td>15.2</td>
</tr>
</tbody>
</table>

4. Similar properties

\[\bullet\] Conversely, the problem is equivalent to finding \(x\) and \(\beta\) satisfying \(\alpha\) and \(\beta\).

\[\bullet\] Extends properties in Buchheim et al. (2008) to lift-and-project formulations, both bringing the support hyperplane method from Veinott [1967] to MIP.

5. RP CGLP is better for multiple root cuts

The feasible set is invariant to \(\rho\), hence changing it only affects cut evaluation.

References


