Branching with Less Repetition

1. Collecting IP Solutions
We can collect near-optimal solutions of an integer program by branching to exhaustion, like in the one-tree approach [Danna et al., 2007] used by CPLEX solver [IBM Corp., 2013]. That entails finding solutions one-by-one, which takes too long if there are too many. Our goal is to remedy that by identifying subproblems with the same completions.

2. Turning a Branching Tree into a Decision Diagram
A bottom-up pass suffices to merge all nodes with isomorphic completions [Bryant, 1986]:
\[ \sum_{j=1}^{n} a_j x_j \leq a_0, \quad a_0, \ldots, a_n \in \mathbb{Z}, \quad (1) \]
we find all values \([\beta, \gamma]\) equivalent to \(a_0\) [Behle, 2007, Abio et al., 2012, Abio and Stuckey, 2014].

3. Equivalence for Single Inequalities
We generalize that to additively separable inequalities with fractional coefficients:
\[ \sum_{j=1}^{n} t_j(x_j) \leq \rho \quad (2) \]
For a partial solution \(\bar{x}_1, \ldots, \bar{x}_{n-k}\), we have
\[ \sum_{j=n-k+1}^{n} t_j(\bar{x}_j) \leq \rho - \sum_{j=1}^{n-k} t_j(\bar{x}_j) \quad (3) \]
Given an unexplored node \(u\) with RHS \(\rho^u\) and an explored node \(v\) with tightest RHS \(\beta^v\):

4. Equivalence for Multiple Inequalities
A partial solution on \(m\) inequalities as those above defines a subproblem of the form
\[ \sum_{j=1}^{n} t_j(x_j) \leq \rho_j - \sum_{j=1}^{n-k} t_j(\bar{x}_j), \quad \forall j = 1, \ldots, m \quad (6) \]
Equivalent RHS values are interdependent, but there is a unique tightest RHS vector:
\[ \sum_{j=1}^{n} t_j(x_j) \leq \rho_j - \sum_{j=1}^{n-k} t_j(\bar{x}_j) \quad (6) \]

5. Results
Our approach is implemented on CPLEX through callbacks. For now, we compare it with one-tree upon fixed variable ordering.

Ours branched less and sometimes ran faster, with geometric means down by 40% and 20%.

6. Next Steps
Find good choices for the \(k\) bottom variables; explore heuristics to choose where to branch next when changing CPLEX default choices.

Extend this approach to the mixed-integer case.

Mix it with branching dominance, which pays off at the top of the branching tree [Fischetti and Toth, 1988, Fischetti and Salvagnin, 2010].

7. References